

# **STRUCTURAL MECHANICS**

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# Chapter 1

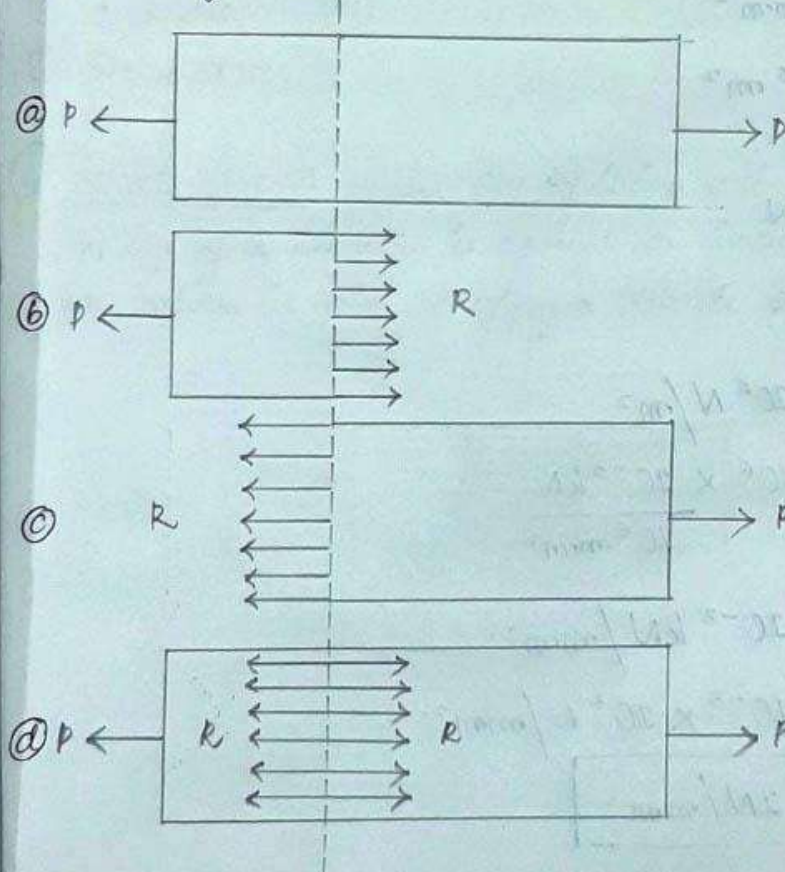
## **SIMPLE AND COMPLEX STRESS AND STRAIN .**

CHAPTER-2

SIMPLE STRESS AND STRAIN:

\* Stress: A resistance force offered by a body against the deformation is called stress.  
(change in shape and size)

- ② The external force acting on the body is called load.
- ③ In a single line, load is applied on the body and stress is induced in the material of the body.
- ④ Stress is denoted by ' $\sigma$ ' (sigma).
- ⑤ Let a rod of uniform cross-sectional area is subjected to pulling force (P), to resist the deformation, then an internal force (R), will be induced inside the material.



$\therefore \text{Stress} = \frac{R}{A}$

$\sigma = \frac{P}{A}$  [∵ R = P]

→ Unit:  $\frac{\text{Newton}}{(\text{metre})^2}$

$= \frac{N}{m^2} = \text{Pascal}$

$R = P$  no tearing of the body

[applied = resisting force force]

Stress =  $\frac{\text{Internal resisting force}}{\text{area}}$

## UNITS OF STRESS

$$1 \text{ Pascal} = 1 \text{ N/m}^2$$

$$1 \text{ Kilopascal} = 10^3 \text{ N/m}^2$$

$$1 \text{ Mega Pascal} = 10^6 \text{ N/m}^2$$

$$1 \text{ Giga Pascal} = 10^9 \text{ N/m}^2$$

$$* 1 \text{ m} \rightarrow 10^3 \text{ mm}$$

$$1 \text{ mm} \rightarrow \frac{1}{10^3} \text{ m} = 10^{-3} \text{ m}$$

$$* 1 \text{ m}^2 = (10^3)^2 \text{ mm}^2 \\ = 10^6 \text{ mm}^2$$

$$* 1 \text{ mm}^2 = 10^{-6} \text{ m}^2$$

$$* 1 \text{ kN} = 10^3 \text{ N}$$

$$* 1 \text{ N} = 10^{-3} \text{ kN}$$

like know,

$$1 \text{ MPa} = 10^6 \text{ N/m}^2$$

$$= \frac{10^6 \times 10^{-3} \text{ kN}}{10^6 \text{ mm}^2}$$

$$= 10^{-3} \text{ kN/mm}^2$$

$$= 10^{-3} \times 10^3 \text{ N/mm}^2$$

$$1 \text{ MPa} = 1 \text{ N/mm}^2$$

$$\text{kilo} = 10^3$$

$$\text{Mega} = 10^6$$

$$\text{Giga} = 10^9$$

$$\text{Tera} = 10^{12}$$

$$\text{Milli} = 10^{-3}$$

$$\text{Micro} = 10^{-6}$$

$$\text{Nano} = 10^{-9}$$

$$\text{Pico} = 10^{-12}$$



\* Strain: This is the ratio of change in dimension to the original dimension.

→ It is denoted by 'e'.

$$\Rightarrow e = \frac{\Delta l}{l} \Rightarrow \frac{\text{change in length}}{\text{original length}} \Rightarrow \frac{\Delta d}{d} \Rightarrow \frac{\Delta v}{v}$$

linear strain
lateral strain
volumetric strain

→ tensile
→ compression

→ Strain has no units i.e. it is unitless.

\* Types of Stress:

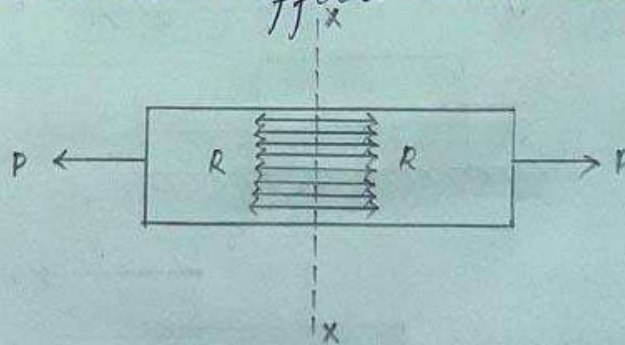
A material is capable of offering three types of stresses:

(i) Tensile stress

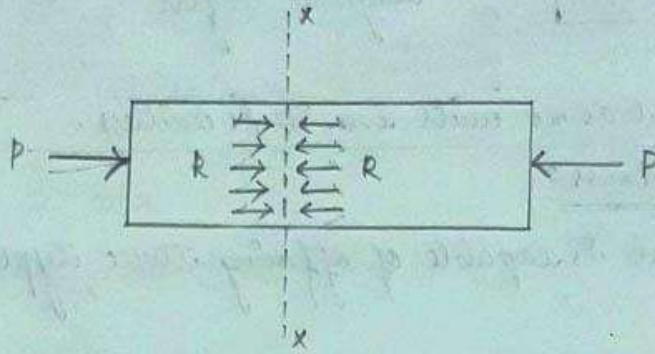
(ii) Compressive stress

(iii) Shear stress

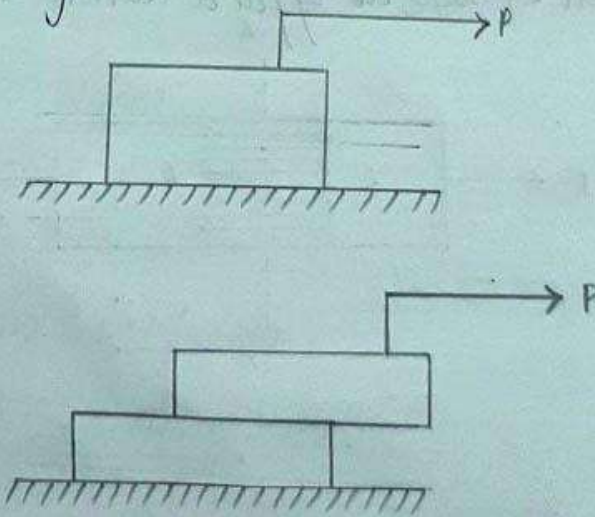
(i) Tensile stress: When the offering resistance by a section of a member is against an increase in length, the section is said to offer a tensile stress.



(ii) Compressive Stress: When the offering resistance by a section of a member is against a decrease in length, the section is said to offer a compressive stress.



(iii) Shear Stress: ( $\tau$ ) A load  $P$  is applied tangentially along the top of the face of the body of which, bottom face is fixed. Such force acting tangentially along a surface is called as shear force.





→ The resistance provided in this case is called shear resistance.

→ Shear stress is defined as the ratio of shear resistance to the shear area.

mathematically, 
$$\tau = \frac{R}{A}$$

\* Types of strain:

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Strain can be of 4 types:

(i) Tensile strain: The ratio of increase in length to its original length, is called tensile strain.

$$e = \frac{\Delta l}{l} \rightarrow \text{here, } \Delta l \rightarrow \text{change in increase in length}$$

(ii) Compressive strain: The ratio of decrease in length to the original length, is called compressive strain.

$$e = \frac{\Delta l}{l} \rightarrow \text{here, } \Delta l \rightarrow \text{change in decrease in length}$$

(iii) Lateral strain: The ratio of the change in lateral dimension to the original lateral dimension, is called lateral strain.

$$e = \frac{\Delta d}{d} \rightarrow \text{here, } \Delta d \rightarrow \text{change in lateral dimension}$$



(iv) Volumetric strain: The ratio of the change in volume to its original volume, is called as volumetric strain.

$$e = \frac{\Delta v}{v} \rightarrow \text{here, } \Delta v \rightarrow \text{change in volume}$$

\* Hooke's Law: <sup>(most imp)</sup>  
<sub>→ marks regular question</sub>

It states that "when a material is loaded such that the intensity of stress within a certain limit, the ratio of the intensity of the stress to the corresponding strain is a constant."

$$\frac{\text{stress}}{\text{strain}} = \frac{\sigma}{e} = \text{constant}$$

$$\Rightarrow \frac{\sigma}{e} = \text{constant}$$

$$\Rightarrow \sigma = \text{constant} \times e$$

$$\Rightarrow \boxed{\sigma \propto e}$$

→ It also can be stated as "Stress is directly proportional to strain, within a certain limit."

\* Modulus of elasticity (or) Young's Modulus:

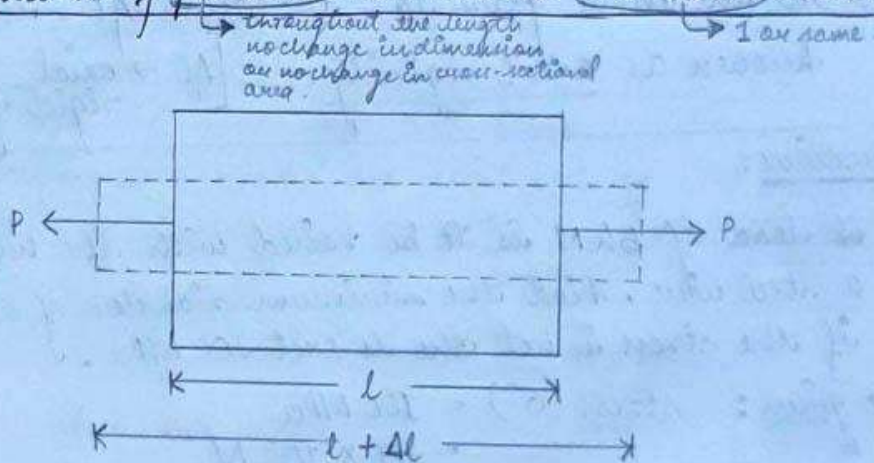
In case of axial loading, the ratio of the intensity of tensile or compressive stress to the corresponding strain is constant.

→ This ratio is called modulus of elasticity or young's modulus and is denoted by 'E'.

$$\frac{\sigma}{e} = \text{constant} = E$$

⇒  $E = \frac{\sigma}{e}$  [∵ constant is nothing but the young's modulus 'E'.]

\* Deformation of prismatic bar due to uni-axial load:



Consider a prismatic bar is subjected to axial tensile load  $P$ . → same as an external force.

The stress will be introduced due to load  $P$  is

given by,  $\sigma = \frac{P}{A}$



we know, strain:  $e = \frac{\Delta l}{l}$

According to Hooke's law,

$$\sigma \propto e$$

$$\Rightarrow \sigma = Ee$$

$$\Rightarrow \frac{P}{A} = E \frac{\Delta l}{l}$$

$$\left[ \begin{array}{l} \because \sigma = \frac{P}{A} \\ \text{and } e = \frac{\Delta l}{l} \end{array} \right]$$

$$\Rightarrow \Delta l = \frac{Pl}{AE}$$

→ In the above formula, the term 'AE' is known as axial rigidity. [AE → axial rigidity]

Questions:

① A load of 5kN is to be raised with the help of a steel wire. Find the minimum diameter of a wire, if the stress is not to exceed 100 MPa.

→ given: stress ( $\sigma$ ) = 100 MPa  
=  $100 \times 10^6 \frac{N}{m^2}$ .

external force (P) = 5kN  
=  $5 \times 10^3 N$





$$\sigma = \frac{P}{A}$$

$$\Rightarrow 100 \times 10^6 = \frac{5 \times 10^3}{\frac{\pi}{4} d^2}$$

$$\Rightarrow d^2 = \frac{5 \times 10^3}{100 \times 10^6 \times \frac{\pi}{4}}$$

$$\Rightarrow d = \sqrt{\frac{5 \times 10^3}{100 \times 10^6 \times \frac{\pi}{4}}}$$

$$\Rightarrow \boxed{d = 7.97 \times 10^{-3} \text{ m}}$$

$$\Rightarrow d = 7.97 \times 10^{-3} \times 10^3 \text{ mm}$$

$$\Rightarrow \boxed{d = 7.97 \text{ mm}}$$

- ② A steel rod 500 mm long and 20 mm x 10 mm in cross-section is subjected to axial pull of 300 kN. If modulus of elasticity is  $2 \times 10^5 \frac{\text{N}}{\text{mm}^2}$ . Calculate the elongation of the rod.  
Also calculate the strain induced in the bar.

→ given:  $l = 500 \text{ mm}$

$$P = 300 \text{ kN} = 300 \times 10^3 \text{ N}$$

$$E = 2 \times 10^5 \text{ N/mm}^2$$

$$A = 20 \text{ mm} \times 10 \text{ mm} \\ = 200 \text{ mm}^2$$

$\Delta l$

$$\therefore \Delta l = \frac{Pl}{AE} = \frac{300 \times 10^3 \times 500}{200 \times 2 \times 10^5} = 3.75 \text{ mm}$$

$$\text{and strain} = \frac{\Delta l}{l} = \frac{3.75}{500} = 7.5 \times 10^{-3}$$

② A hollow cylinder 2 m long has an outside diameter of 50 mm and inside diameter of 30 mm. If the cylinder is carrying a load of 25 kN, find the stress in cylinder, also find deformation of the cylinder. Take  $E = 100 \text{ GPa}$ .

→ given:  $l = 2 \text{ m} = 2 \times 1000 \text{ mm}$   
 $= 2000 \text{ mm}$

$$D = 50 \text{ mm}$$

$$d = 30 \text{ mm}$$

$$E = 100 \text{ GPa}$$

$$P = 25 \text{ kN}$$
  
 $= 25 \times 10^3 \text{ N}$

∴ Area of hollow cylinder

$$A = \frac{\pi}{4} (D^2 - d^2)$$

$$\Rightarrow A = \frac{\pi}{4} (50^2 - 30^2)$$

$$\Rightarrow \boxed{A = 1256 \text{ mm}^2}$$



$$\begin{aligned}
 E &= 100 \text{ GPa} \\
 &= 100 \times 10^9 \frac{\text{N}}{\text{m}^2} \\
 &= 100 \times 10^9 \frac{\text{N}}{10^6} \\
 &= 100 \times 10^3 \frac{\text{N}}{\text{mm}^2} \\
 E &= 100 \times 10^3 \frac{\text{N}}{\text{mm}^2}
 \end{aligned}$$

$$\begin{aligned}
 1 \text{ m} &\rightarrow 10^3 \text{ mm} \\
 1 \text{ m}^2 &\rightarrow (10^3)^2 \text{ mm}^2 \\
 &\rightarrow 10^6 \text{ mm}^2
 \end{aligned}$$

$$\therefore \sigma = \frac{P}{A} = \frac{25 \times 10^3}{1256} = 19.90 \frac{\text{N}}{\text{mm}^2}$$

$$\text{and } \Delta l = \frac{Pl}{AE} = \frac{25 \times 10^3 \times 2000}{1256 \times 100 \times 10^3} = 0.398 \text{ mm}$$

### \* POISSON'S RATIO $\left(\mu / \frac{1}{m}\right)$

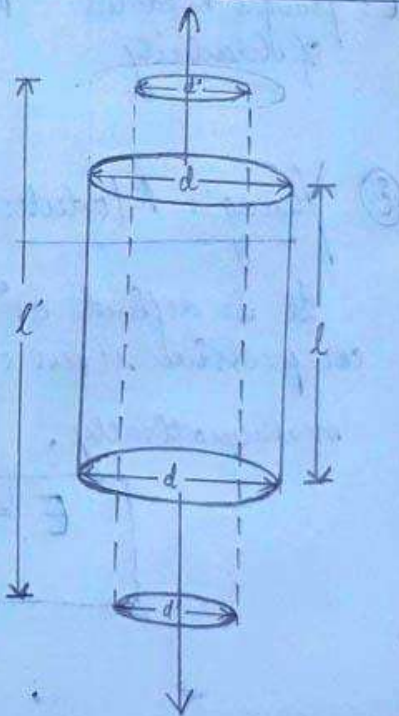
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$$\text{Linear strain} = + \frac{l'}{l}$$

$$\text{Lateral strain} = - \frac{d'}{d}$$

→ Poisson's ratio is defined as "the ratio between the lateral strain to the longitudinal strain."

→ It is denoted as  $\mu$  or  $\frac{1}{m}$  and the value of  $\mu$  is always less than 1.



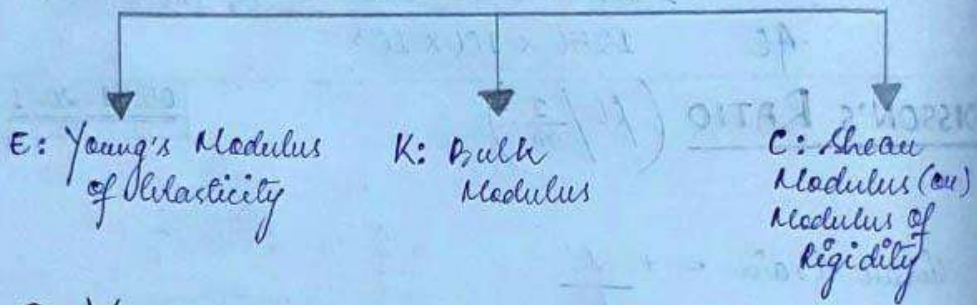


mathematically,

$$\mu = \frac{- \text{lateral strain}}{\text{longitudinal strain}}$$

- It is a constant and it has no unit.
- ex: (i) Poisson ratio of concrete = 0.15 to 0.3
- (ii) Poisson ratio of steel = 0.27 to 0.3
- Poisson ratio varies from -1 to 0.5

### Elastic Constant



#### (i) Young's Modulus (E):

It is defined as "the ratio of tensile or compressive stress to the corresponding strain."

mathematically,

$$E = \frac{\sigma}{e}$$

(i) Bulk Modulus (K) :

When an uniform element is subjected to equal stress in 3 mutually perpendicular directions then "the ratio of direct stress to the volumetric strain is known as bulk modulus."

mathematically,

$$K = \frac{\sigma}{e_v}$$

$$\left[ \because e_v = \frac{\Delta v}{v} \right]$$

volumetric strain

(ii) Shear Modulus (C or G) :

It is defined as "the ratio of shear stress to the shear strain."

→ Higher the value of shear modulus, shows that the body is highly rigid.

mathematically,

$$C = \frac{\tau}{e}$$

where,  $\tau \rightarrow$  shear stress  
 $e \rightarrow$  shear strain



\* Relationship between Elastic Constants E, K, C:

(i) Relationship between E and K:

$$E = 3K \left( 1 - \frac{2}{m} \right)$$

(ii) Relationship between E and C:

$$E = 2C \left( 1 + \frac{1}{m} \right)$$

where,  
 $\frac{1}{m}$  is poisson's  
ratio

(iii) Relationship between E, K and C:

$$E = \frac{9KC}{3K + C}$$



\* Proof:

$$E = \frac{9KC}{3K+C}$$

we know,

$$E = 3K \left(1 - \frac{2}{M}\right) \Rightarrow \boxed{\frac{1-2}{M} = \frac{E}{3K}} \quad \text{--- ①}$$

$$E = 2C \left(1 + \frac{1}{m}\right) \Rightarrow \boxed{\frac{1+1}{m} = \frac{E}{2C}} \quad \text{--- ②}$$

multiplying 2 in eqn ②

$$2 \left(1 + \frac{1}{M}\right) = \cancel{2} \left(\frac{E}{\cancel{2}C}\right)$$

$$\Rightarrow \boxed{2 + \frac{2}{M} = \frac{E}{C}} \quad \text{--- ③}$$

now adding ① and ③

$$1 - \frac{\cancel{2}}{M} + 2 + \frac{\cancel{2}}{M} = \frac{E}{3K} + \frac{E}{C}$$

$$\Rightarrow 1 + 2 = \frac{E}{3K} + \frac{E}{C}$$

$$\Rightarrow 3 = \frac{EC + E3K}{3KC}$$

$$\Rightarrow 3(3KC) = EC + E3K$$

$$\Rightarrow 9KC = E(C + 3K)$$

$$\Rightarrow \boxed{E = \frac{9KC}{3K + C}} \quad \text{hence proved.}$$

Questions:

① The modulus of rigidity of a material is  $0.8 \times 10^5 \frac{N}{mm^2}$ . Find the poisson's ratio if the modulus of elasticity of that material is  $2.1 \times 10^5 \frac{N}{mm^2}$ .

$$\rightarrow \text{given: } C = 0.8 \times 10^5 \frac{N}{mm^2}$$

$$E = 2.1 \times 10^5 \frac{N}{mm^2}$$

$$\mu \text{ or } \frac{1}{M} = ?$$

$$\therefore E = \left\{ 2C \left( 1 + \frac{1}{M} \right) \right.$$

$$\Rightarrow 2.1 \times 10^5 \frac{N}{mm^2} = 2 \times 0.8 \times 10^5 \frac{N}{mm^2} \left( 1 + \frac{1}{M} \right)$$

$$\Rightarrow \frac{2.1 \times 10^5}{2 \times 0.8 \times 10^5} = 1 + \frac{1}{M}$$

$$\Rightarrow \frac{2.1 \times 10^5}{2 \times 0.8 \times 10^5} - 1 = \frac{1}{M}$$

$$\Rightarrow \frac{1}{M} \text{ or } \mu = \frac{2.1 \times 10^5}{2 \times 0.8 \times 10^5} - 1.$$

$$\Rightarrow \boxed{\mu = 0.3125} \text{ Ans}$$



② The modulus of rigidity of material is  $0.8 \times 10^5 \frac{N}{mm^2}$ . When a  $6\text{mm} \times 6\text{mm}$  rod of this material was subjected to an axial pull of  $3600\text{ N}$ . It was found that the lateral dimension of the rod changed to  $5.9991\text{ mm} \times 5.9991\text{ mm}$ . Find the poisson's ratio and modulus of elasticity of that rod.

→ given:  $C = 0.8 \times 10^5 \frac{N}{mm^2}$ .

$$A = 6\text{ mm} \times 6\text{ mm} \\ = 36\text{ mm}^2$$

$$P = 3600\text{ N}$$

$$\sigma = \frac{P}{A} = \frac{3600}{36} = 100 \frac{N}{mm^2}$$

longitudinal strain,  $E = \frac{\sigma}{e} \Rightarrow \boxed{e = \frac{\sigma}{E}}$

$$\text{lateral strain} = \frac{\text{change in lateral dimension}}{\text{original dimension}}$$

$$= \frac{6 - 5.9991}{6} = 0.00015$$



Poisson's ratio:

$$\frac{1}{M} = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$$\Rightarrow \frac{1}{M} = \frac{0.00015}{\frac{\sigma}{E}}$$

$$\Rightarrow \frac{1}{M} = \frac{0.00015}{100} \times E$$

$$\Rightarrow \frac{1}{EM} = \frac{0.00015}{100}$$

$$\Rightarrow ME = \frac{100}{0.00015}$$

$$\Rightarrow \boxed{ME = \frac{2 \times 10^6}{3}} \quad \text{--- (1)}$$

we know,  $E = 2C \left(1 + \frac{1}{M}\right)$

$$\Rightarrow ME = 2C \left(\frac{M+1}{M}\right)$$

$$\Rightarrow ME = 2C (M+1)$$

$$\Rightarrow \boxed{ME = 2 \times 0.8 \times 10^5 (M+1)} \quad \text{--- (2)}$$

equating eq<sup>n</sup> (1) and (2)

$$\Rightarrow 2 \times 0.8 \times 10^5 (M+1) = \frac{2 \times 10^6}{3}$$

$$\Rightarrow M + 1 = \frac{2 \times 10^6}{3(2 \times 0.8 \times 10^5)}$$

$$\Rightarrow M = \frac{2 \times 10^6}{3(2 \times 0.8 \times 10^5)} - 1$$

$$\Rightarrow \boxed{M = 3.167}$$

$$\therefore \mu = \frac{1}{M} = \frac{1}{3.167}$$

$$\Rightarrow \boxed{\mu = 0.315}$$

$$\therefore E = 2C \left(1 + \frac{1}{M}\right)$$

$$\Rightarrow E = 2 \times 0.8 \times 10^5 (1 + 0.315)$$

$$\Rightarrow E = 210400$$

$$\Rightarrow \boxed{E = 2.1 \times 10^5 \frac{N}{\text{mm}^2}}$$

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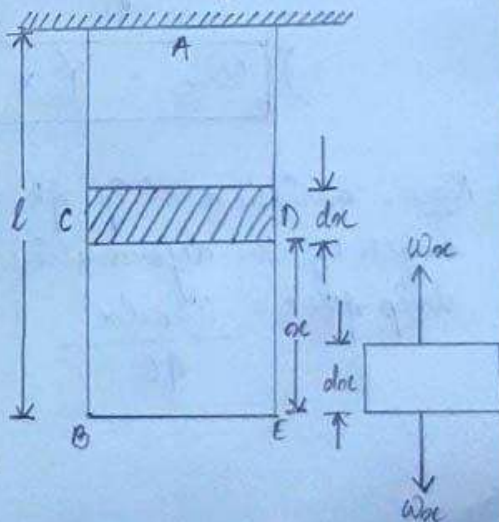
\* Axial deformation of bar due to its self-weight:

Considering a prismatic bar of uniform cross-sectional area of length 'l' suspended freely.

weight of the bar =  $w$

unit weight of the bar =  $\gamma$

$$\left[ \begin{array}{l} \gamma = \frac{w}{\text{volume}} \\ \Rightarrow \gamma = \frac{w}{Ax} \end{array} \right. \begin{array}{l} A \rightarrow \text{area} \\ x \rightarrow \text{length} \\ Ax \rightarrow \text{volume} \end{array}$$





Assuming cross-sectional area = 'A'  
 Consider a small element of length 'dx' at a distance 'x' from the free end.

The element is subjected to weight of bar below element (i.e. BCDE)

weight of bar below element dx  
 $\Rightarrow W_{dx} = \gamma (A x)$

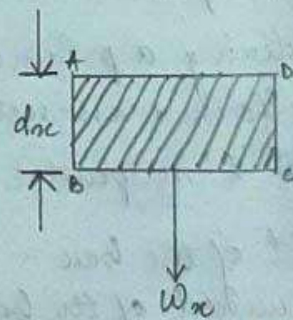
Elongation of elemental length dx  
 $\Rightarrow \boxed{d\Delta = \frac{W_{dx} dx}{AE}}$

let  $E, \rho, \gamma$ , be the Young's Modulus, density and unit weight of the body respectively.

we know, (volume)  $V_x = A \times x$

$$\gamma = \frac{W_x}{V_x}$$

$$\Rightarrow \boxed{W_x = \gamma \times V_x}$$



Now, consider a strip ABCD of length by the deformation for the strip ABCD =  $\frac{W_x dx}{AE}$



$$\text{Total deformation of the bar} = \int_0^l \frac{Wx \times dx}{AE}$$

$$= \int_0^l \frac{Y \times Vx}{AE} dx$$

$$= \int_0^l \frac{Y \times A \times \alpha}{AE} dx$$

$$= \int_0^l \frac{Y \alpha}{E} dx$$

$$= \frac{Y}{E} \int_0^l \alpha dx \quad \left[ \because Y \text{ and } E \text{ are constants} \right]$$

$$= \frac{Y}{E} \left( \frac{\alpha^2}{2} \right)_0^l$$

$$= \frac{Y}{E} \left[ \frac{l^2}{2} - \frac{0^2}{2} \right]$$

$$= \frac{Y}{E} \times \frac{l^2}{2}$$

$$\Rightarrow \Delta = \frac{Y l^2}{2E}$$

$\Delta l \rightarrow$  total deformation of the bar.

\* **NOTES:**

- ① Ultimate stress =  $\frac{\text{maximum load}}{\text{Area}}$
- ② Yield stress =  $\frac{\text{yield point load}}{\text{Area}}$
- ③ Safe stress =  $\frac{\text{yield stress}}{\text{load factor}}$

Questions:

- ① A following data refers to a mild steel specimen tested in a laboratory:
  - (i) diameter of specimen = 25 mm
  - (ii) length of specimen = 300 mm
  - (iii) extension under a load 50 kN = 0.045 mm
  - (iv) load at yield point = 127.65 kN
  - (v) maximum load = 208.60 kN
  - (vi) length of the specimen after failure = 375 mm
  - (vii) neck diameter = 17.75 mm

Now, determine: (i) Young's Modulus  
 (ii) Yield point stress  
 (iii) Ultimate stress  
 (iv) Safe stress adopting a factor of safety of 2.



→ given:  $d = 25 \text{ mm}$   
 $l = 300 \text{ mm}$   
 $P = 50 \text{ kN}$   
 $= 50 \times 10^3 \text{ N}$   
 $\Delta l = 0.045 \text{ mm}$

yield point load =  $127.65 \text{ kN}$   
 $= 127.65 \times 10^3 \text{ N}$

maximum load =  $208.60 \text{ kN}$   
 $= 208.60 \times 10^3 \text{ N}$

load factor = 2.

(i)  $\Delta l = \frac{Pl}{AE}$

$\Rightarrow E = \frac{Pl}{A\Delta l}$

$\Rightarrow E =$

$$\begin{aligned}
 \textcircled{\text{ii}} \text{ Yield point stress} &= \frac{\text{yield point load}}{\text{area}} \\
 &= \frac{127.65 \times 10^3 \text{ N}}{\frac{\pi}{4} (25)^2 \text{ mm}^2} \\
 &= 260.05 \frac{\text{N}}{\text{mm}^2}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{\text{iii}} \text{ Ultimate stress} &= \frac{\text{maximum load}}{\text{area}} \\
 &= \frac{208.60 \times 10^3 \text{ N}}{\frac{\pi}{4} (25)^2 \text{ mm}^2} \\
 &= 424.96 \frac{\text{N}}{\text{mm}^2}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{\text{iv}} \text{ Safe stress} &= \frac{\text{yield stress}}{\text{load factor}} \\
 &= \frac{260.05}{2} \frac{\text{N}}{\text{mm}^2} \\
 &= 130.025 \frac{\text{N}}{\text{mm}^2}
 \end{aligned}$$



# ***Mechanical Properties Of Materials***

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## MECHANICAL PROPERTIES OF MATERIALS :

### ① Rigidity:

- ① This is defined as the property possessed by a solid body to change its shape.
- ② It means when an external force is applied to the solid material, there won't be any change in its shape due to intermolecular attraction by the closely packed particles.
- ③ This is the property of the material to resist the bending.

### ② Elasticity:

- ① This is the property of a body by virtue of which it returns its original shape after removal of external force causing deformation which is applied on it.
- ② Elastic property entirely depends upon the type of material and not on the shape and size.

### ③ Plasticity:

- ① This is the ability of a solid material to undergo permanent deformation when force is applied to it.
- ② The ability of a material to retain the changed shape under application of load is known as plasticity.
- ③ Plastic deformation is the property of ductile and malleable solids.



#### ④ Compressibility:

- ① This is the property of material by virtue of which it tends to flatten and reduce in size under pressure.
- ② This nature or property of material changes the inter-molecular structure of the material.

#### ⑤ Hardness:

- ① The property of material by virtue of which it resists the local surface deformation when undergoes abrasion, drilling, impact, etc.
- ② It is the state of material, being hard for which it can withstand friction.

#### ⑥ Toughness:

- ① The amount of energy per unit volume that a material can absorb before rupture is called toughness.
- ② It can be defined as the ability of a material to resist breaking when force is applied to it.
- ③ This property allows the material to deform before rupture or fracture.

#### ⑦ Stiffness:

- ① The property of material which resists deformation when a force is applied to it. This is the rigidity of a material.
- ② The material having more flexibility has less stiffness.
- ③ A stiff material has high Young modulus.



### 8) Brittleness:

- (i) The property of a material by which it fractures when subjected to stress without deformation.
- (ii) Brittle material has a little tendency to deform before rupture.
- (iii) It has small plastic region.

ex: Bone, concrete, ceramic, cast iron, glass products, etc.

### 9) Ductility:

- (i) It is defined as the ability of a material to undergo permanent deformation through elongation and reduction in cross-sectional area or bending at room temperature without fracturing.
- (ii) This is an ability to undergo last permanent deformation in tension.

Example of ductile material → copper, aluminium, steel.

★ Opposite to ductile is brittle.

### 10) Malleability:

- (i) This is the property of material by which it can be beaten to form its thin sheets.

ex: lead, tin, gold, silver, aluminium, copper, iron.

### 11) Creep:

- (i) This is the permanent change in shape and size of a material which increases as a function of time under application of load and elevated temperature.
- (ii) Creep is time independent.
- (iii) Creep begins at different temperatures for different material.



### 12. Fatigue:

- (i) This is the deterioration of the material subjected to repeated cycle of stress and strain resulting in progressive cracking, eventually producing fracture.
- (ii) This is the weakening of a material caused by cyclic loading that results in progressive damage and the growth of cracks.
- (iii) This is responsible for 90% of mechanical failure.

### 13. Durability:

- (i) The ability of a material to remain serviceable during the useful time without damaging the material.
- (ii) It represents how long the material works.

### 14. Tenacity:

→ The property of material to resist the breaking is known as tenacity.

### ★ NOTES:

#### ① Percentage of Elongation:

(a) It is a measure of ductility.

(b) This can be obtained as = 
$$\frac{\text{final length} - \text{initial length}}{\text{initial length}} \times 100$$

$$\text{or } \frac{\Delta l}{l} \times 100$$

where,  $\Delta l$  = change in length.



## ② Percentage of reduction in area:

(a) This is the measure of the specimen that how much that specimen narrowed when it undergoes some load application.

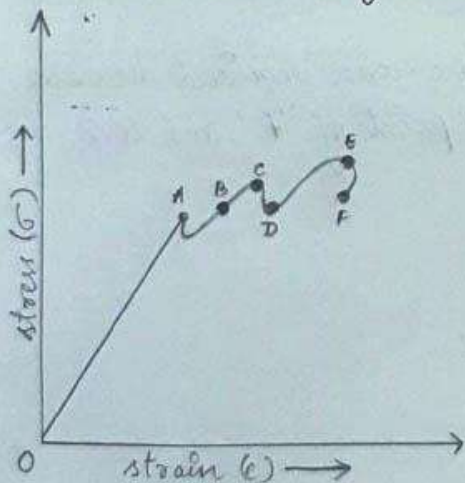
(b) It is obtained as follows:

$$\frac{\text{final area} - \text{initial area}}{\text{initial area}} \times 100$$

## \* Signification of Percentage elongation and Percentage Reduction in area:

- (i) Percentage elongation is a property of material which provides a value of its ductility.
- (ii) The greater deformation before breaking shows the material is more ductile.
- (iii) Percentage elongation is a measure of ductility.
- (iv) Percentage reduction area is also a measure of ductility.

## \* Stress - Strain diagram for mild steel: (Imp)



→ This curve is obtained when a mild steel specimen undergoes a tensile test. The plot from O to A is a straight line, this portion obeys Hooke's law and the straight line is called the limit of proportionality.

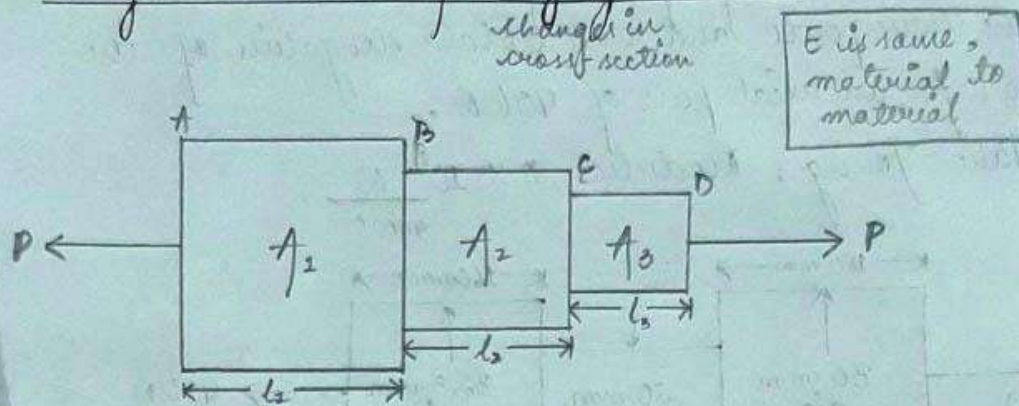
→ In this range of extension, the stress is proportional to strain i.e.  $\sigma \propto \epsilon$



- If the specimen is extended beyond the limit of proportionality up to the point 'B', the material still remains elastic.
- But from A to B, stress-strain relation is not linear.
- The stress at 'B' is called elastic limit.
- Again if the specimen is extended beyond the elastic limit, plastic deformation occurs.
- In the range of 'B' to 'C' strain increases without increase in stress.
- At point 'C' the material goes extended with decrease in load and the stress at point C is called upper-yield point.
- At point 'D' the material again offers resistance to greater extension and the stress corresponding to this point is called lower-yield point.
- As the load is increased, the extension increases and the point 'E' indicates the necking of the specimen and the stress corresponding to this point is called ultimate tensile stress.
- As the extension is increased, the load required decreases and the specimen breaks at the point of 'F' and this point is called stress failure.

\* Elongation in bar of varying section:

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The elongation for the portion AB:  $\Delta l_1 = \frac{Pl_1}{A_1 E}$

The elongation for the portion BC:  $\Delta l_2 = \frac{Pl_2}{A_2 E}$

The elongation for the portion CD:  $\Delta l_3 = \frac{Pl_3}{A_3 E}$

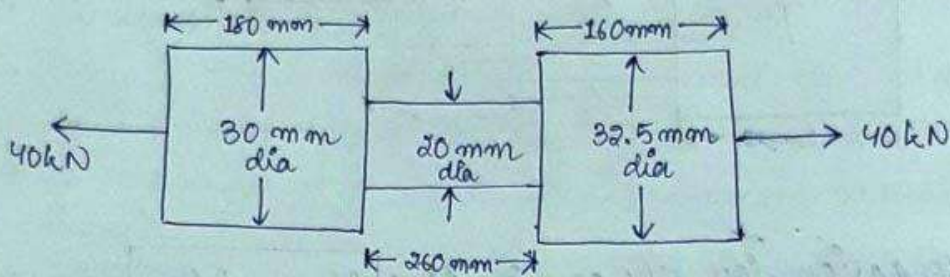
So total elongation:  $\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3$

$$= \frac{Pl_1}{A_1 E} + \frac{Pl_2}{A_2 E} + \frac{Pl_3}{A_3 E}$$

$$= \frac{P}{E} \left( \frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3} \right)$$



- ① A bar consisting of 3 lengths. Find the stresses in the three parts and the total elongation of the bar for an axial pull of 40 kN. Take Young's Modulus  $2 \times 10^5 \frac{N}{mm^2}$ .



→ given:  $P = 40 \text{ kN} = 40 \times 10^3 \text{ N}$   
 $E = 2 \times 10^5 \frac{N}{mm^2}$

$$A_1 = \frac{\pi}{4} (30)^2 = 706.85 \text{ mm}^2$$

$$A_2 = \frac{\pi}{4} (20)^2 = 314.15 \text{ mm}^2$$

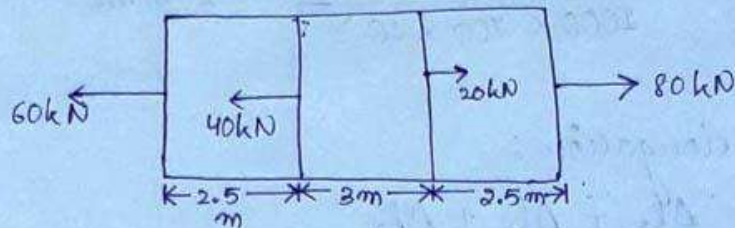
$$A_3 = \frac{\pi}{4} (32.5)^2 = 829.579 \text{ mm}^2$$

$$\Delta l_1 = \frac{40 \times 10^3 \times 180}{706.85 \times 2 \times 10^5} = 0.50 \text{ mm}$$

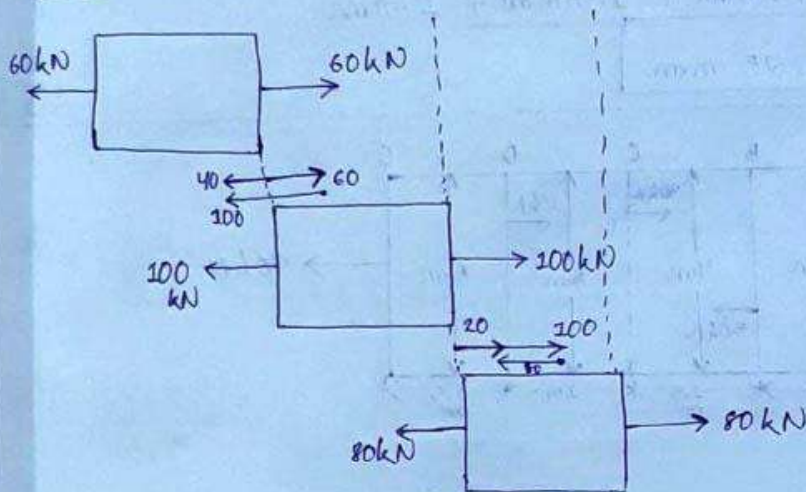
$$\Delta l_2 = \frac{40 \times 10^3 \times 260}{314.15 \times 2 \times 10^5} = 0.165 \text{ mm}$$

$$\Delta l_3 = \frac{40 \times 10^3 \times 160}{829.579 \times 2 \times 10^5} = 0.038 \text{ mm}$$

③ A steel rod ABCD of  $1000 \text{ mm}^2$  cross-sectional area and  $8 \text{ m}$  long is subjected to forces as shown in the figure below. Find the total deformation if young's modulus of the bar is  $200 \text{ GPa}$ . All forces are in  $\text{kN}$ .



FBD:



$$P_1 = 60 \text{ kN} = 60 \times 10^3 \text{ N}$$

$$P_2 = 100 \text{ kN} = 100 \times 10^3 \text{ N}$$

$$P_3 = 80 \text{ kN} = 80 \times 10^3 \text{ N}$$

$$l_1 = 2.5 \text{ m} = 2.5 \times 10^3 \text{ mm}$$

$$l_2 = 3 \text{ m} = 3 \times 10^3 \text{ mm}$$

$$l_3 = 2.5 \text{ m} = 2.5 \times 10^3 \text{ mm}$$

$$E = 200 \text{ GPa}$$

$$= 200 \times 10^9 \frac{\text{N}}{\text{m}^2}$$

$$= 200 \times 10^9 \frac{\text{N}}{10^6 \text{ mm}^2}$$

$$= 200 \times 10^{9-6} \frac{\text{N}}{\text{mm}^2}$$

$$= 200 \times 10^3 \frac{\text{N}}{\text{mm}^2}$$

$$A = 1000 \text{ mm}^2$$



$$\Delta l_1 = \frac{P_1 l_1}{AE} = \frac{60 \times 10^3 \times 2.5 \times 10^3}{1000 \times 200 \times 10^3} = 0.75 \text{ mm}$$

$$\Delta l_2 = \frac{P_2 l_2}{AE} = \frac{100 \times 10^3 \times 3 \times 10^3}{1000 \times 200 \times 10^3} = 1.5 \text{ mm}$$

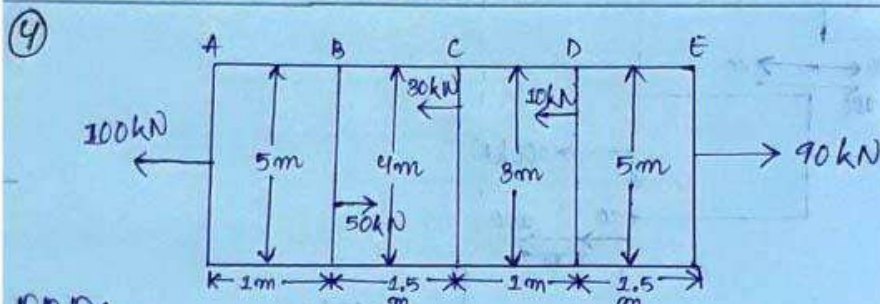
$$\Delta l_3 = \frac{P_3 l_3}{AE} = \frac{80 \times 10^3 \times 2.5 \times 10^3}{1000 \times 200 \times 10^3} = 1 \text{ mm}$$

∴ Total elongation:

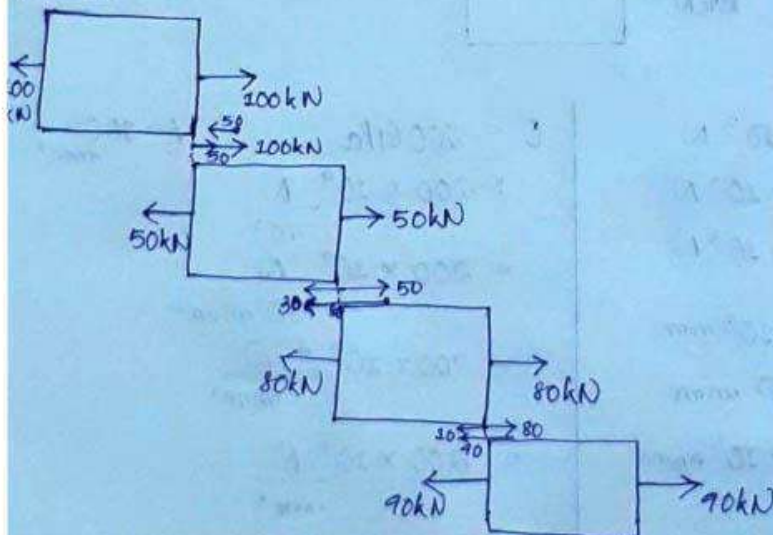
$$\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3$$

$$\Rightarrow \Delta l = 0.75 \text{ mm} + 1.5 \text{ mm} + 1 \text{ mm}$$

$$\Rightarrow \Delta l = 3.25 \text{ mm}$$



FBD:



$$P_1 = 100 \text{ kN} = 100 \times 10^3 \text{ N}$$

$$P_2 = 50 \text{ kN} = 50 \times 10^3 \text{ N}$$

$$P_3 = 80 \text{ kN} = 80 \times 10^3 \text{ N}$$

$$P_4 = 90 \text{ kN} = 90 \times 10^3 \text{ N}$$

$$l_1 = 1 \text{ m}$$

$$l_2 = 1.5 \text{ m}$$

$$l_3 = 1 \text{ m}$$

$$l_4 = 1.5 \text{ m}$$

$$d_1 = 5 \text{ m}$$

$$d_2 = 4 \text{ m}$$

$$d_3 = 3 \text{ m}$$

$$d_4 = 5 \text{ m}$$

$$A_1 = \frac{\pi}{4} (5)^2 = 19.63 \text{ m}^2$$

$$A_2 = \frac{\pi}{4} (4)^2 = 12.56 \text{ m}^2$$

$$A_3 = \frac{\pi}{4} (3)^2 = 7.06 \text{ m}^2$$

$$A_4 = \frac{\pi}{4} (5)^2 = 19.63 \text{ m}^2$$

$$E = 2.5 \text{ MPa}$$

$$= 2.5 \times 10^6 \frac{\text{N}}{\text{m}^2}$$

$$\Delta l_1 = \frac{P_1 l_1}{A_1 E} = \frac{100 \times 10^3 \times 1}{19.63 \times 2.5 \times 10^6} = 2.037 \times 10^{-3} \text{ m}$$

$$\Delta l_2 = \frac{P_2 l_2}{A_2 E} = \frac{50 \times 10^3 \times 1.5}{12.56 \times 2.5 \times 10^6} = 2.38 \times 10^{-3} \text{ m}$$

$$\Delta l_3 = \frac{P_3 l_3}{A_3 E} = \frac{80 \times 10^3 \times 1}{7.06 \times 2.5 \times 10^6} = 4.53 \times 10^{-3} \text{ m}$$

$$\Delta l_4 = \frac{P_4 l_4}{A_4 E} = \frac{90 \times 10^3 \times 1.5}{19.63 \times 2.5 \times 10^6} = 2.75 \times 10^{-3} \text{ m}$$

∴ Total elongation:

$$\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3 + \Delta l_4$$

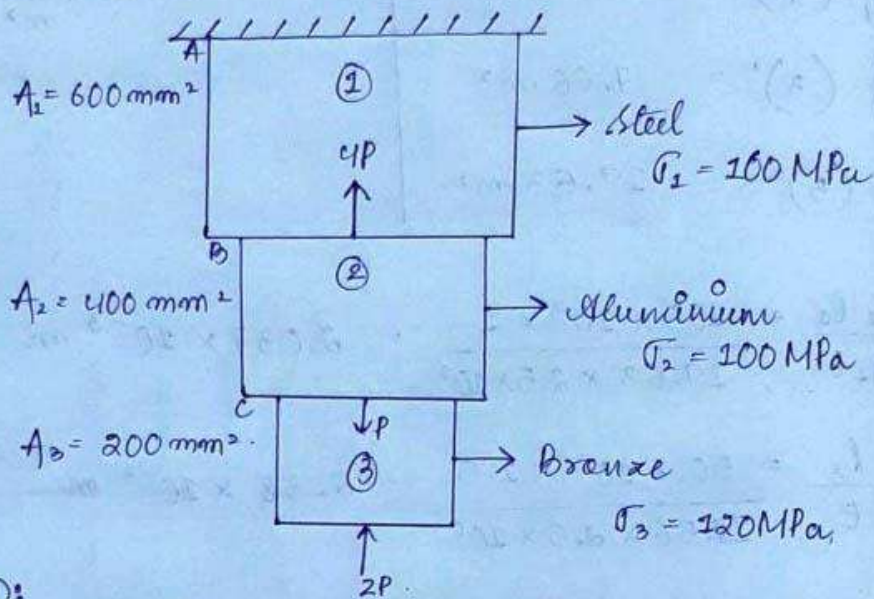
$$\Rightarrow \Delta l = (2.037 \times 10^{-3}) + (2.38 \times 10^{-3}) + (4.53 \times 10^{-3}) + (2.75 \times 10^{-3}) \text{ m}$$

$$\Rightarrow \Delta l = 0.011697 \text{ m}$$

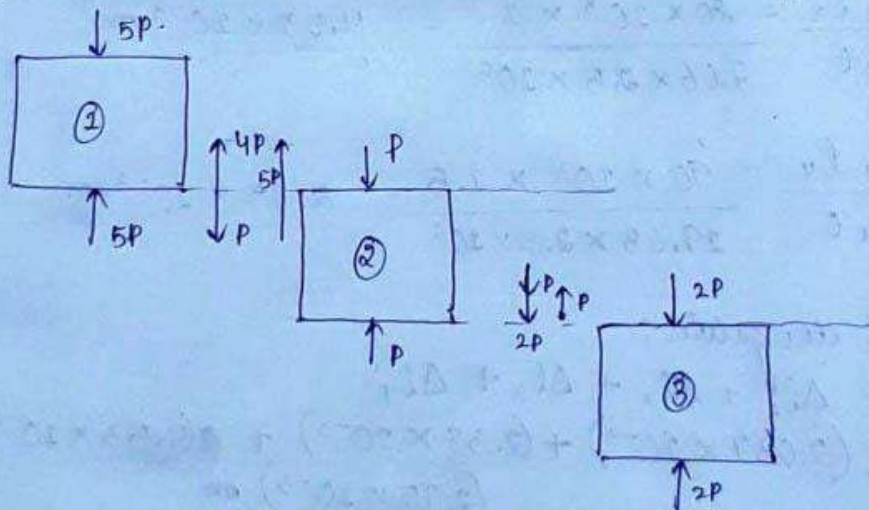
$$\Rightarrow \Delta l = 11.697 \text{ mm}$$



- ⑤ An aluminium bar is rigidly attached between steel bar and a bronze bar shown in figure. Axial forces are applied at the position indicated. Determine the maximum value of  $P$  that will not exceed stress of 160 MPa in steel, 100 MPa in aluminium and 120 MPa in bronze.



FBD:



$$\sigma_1 = 160 \text{ MPa} = 160 \times 10^6 \frac{\text{N}}{\text{m}^2} = 160 \times \frac{10^6 \text{ N}}{10^6 \text{ mm}^2}$$

$$\Rightarrow \boxed{\sigma_1 = 160 \frac{\text{N}}{\text{mm}^2}}$$

$$\sigma_2 = 100 \text{ MPa} = 100 \times 10^6 \frac{\text{N}}{\text{m}^2} = 100 \times \frac{10^6 \text{ N}}{10^6 \text{ mm}^2}$$

$$\Rightarrow \boxed{\sigma_2 = 100 \frac{\text{N}}{\text{mm}^2}}$$

$$\sigma_3 = 120 \text{ MPa} = 120 \times 10^6 \frac{\text{N}}{\text{m}^2} = 120 \times \frac{10^6 \text{ N}}{10^6 \text{ mm}^2}$$

$$\Rightarrow \boxed{\sigma_3 = 120 \frac{\text{N}}{\text{mm}^2}}$$

$$A_1 = 600 \text{ mm}^2$$

$$P_1 = 5P$$

$$A_2 = 400 \text{ mm}^2$$

$$P_2 = P$$

$$A_3 = 200 \text{ mm}^2$$

$$P_3 = 2P$$

$\therefore$  we know that:

for section ①:

$$\sigma_1 = \frac{P_1}{A_1}$$

$$\Rightarrow 160 = \frac{5P}{600} \Rightarrow P_1 = \frac{160 \times 600}{5}$$

$$\Rightarrow \boxed{P_1 = 19200 \text{ N}}$$



for section (2) :

$$\sigma_2 = \frac{P_2}{A_2}$$

$$\Rightarrow 100 = \frac{P_2}{400} \Rightarrow P_2 = 100 \times 400$$

$$\Rightarrow \boxed{P_2 = 40000 \text{ N}}$$

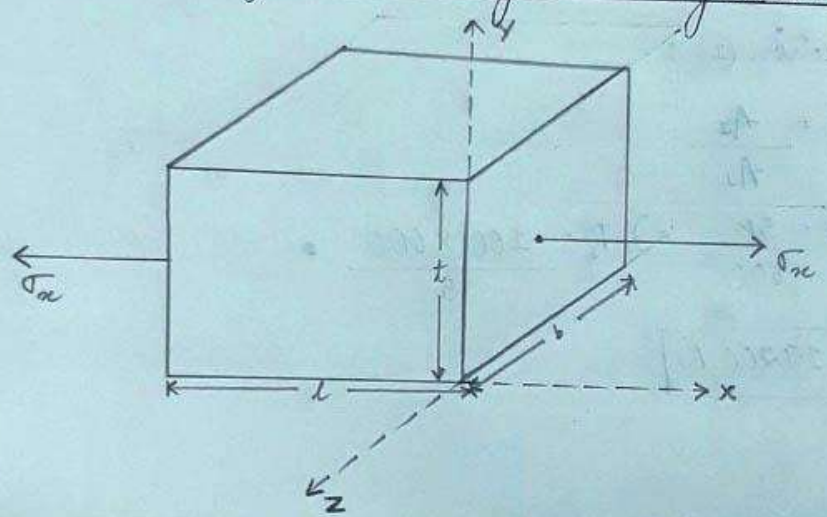
for section (3) :

$$\sigma_3 = \frac{P_3}{A_3}$$

$$\Rightarrow 120 = \frac{P_3}{200} \Rightarrow P_3 = \frac{120 \times 200}{1}$$

$$\Rightarrow \boxed{P_3 = 12000 \text{ N}}$$

27.11.2021  
① Volumetric strain of a rectangular section Under the action of a stress along the longitudinal strain:





Let  $l, b, t$  are the length, breadth and thickness of the rectangular block.

$\sigma_x$  = tensile stress in  $x$ -direction

$E$  = Young's Modulus

$\mu$  = Poisson's ratio

$$= \frac{-\text{lateral strain}}{\text{longitudinal strain}}$$

longitudinal strain :  $\boxed{e_x = \frac{\sigma_x}{E}}$

$$\mu = -\frac{e_y}{e_x}$$

$$\Rightarrow e_y = -\mu \cdot e_x$$

$$\Rightarrow \boxed{e_y = -\mu \frac{\sigma_x}{E}}$$

$$\left[ \because e_x = \frac{\sigma_x}{E} \right]$$

Similarly,  $\boxed{e_z = -\mu \frac{\sigma_x}{E}}$

But we know that,

$$\boxed{e_x = \frac{\delta l}{l}}$$

$$, \quad \boxed{e_y = \frac{\delta b}{b}}$$

$$, \quad \boxed{e_z = \frac{\delta t}{t}}$$

We know that,

volume of a rectangular <sup>(3D)</sup> block is given by:

$$V = l \times b \times t$$

Now, differentiating both the sides:

$$\frac{dV}{V} = \frac{dl}{l} + \frac{db}{b} + \frac{dt}{t}$$

$$\begin{aligned} \therefore \frac{d(l \times b \times t)}{l} + \frac{d(l \times b \times t)}{b} + \frac{d(l \times b \times t)}{t} \\ = \frac{dl}{l} + \frac{db}{b} + \frac{dt}{t} \end{aligned}$$

$$\Rightarrow e_v = e_x + e_y + e_z$$

$$\begin{aligned} \therefore e_v = \frac{dV}{V} &= \text{volumetric strain} \\ e_x &= \frac{dl}{l} & e_z &= \frac{dt}{t} \\ e_y &= \frac{db}{b} \end{aligned}$$



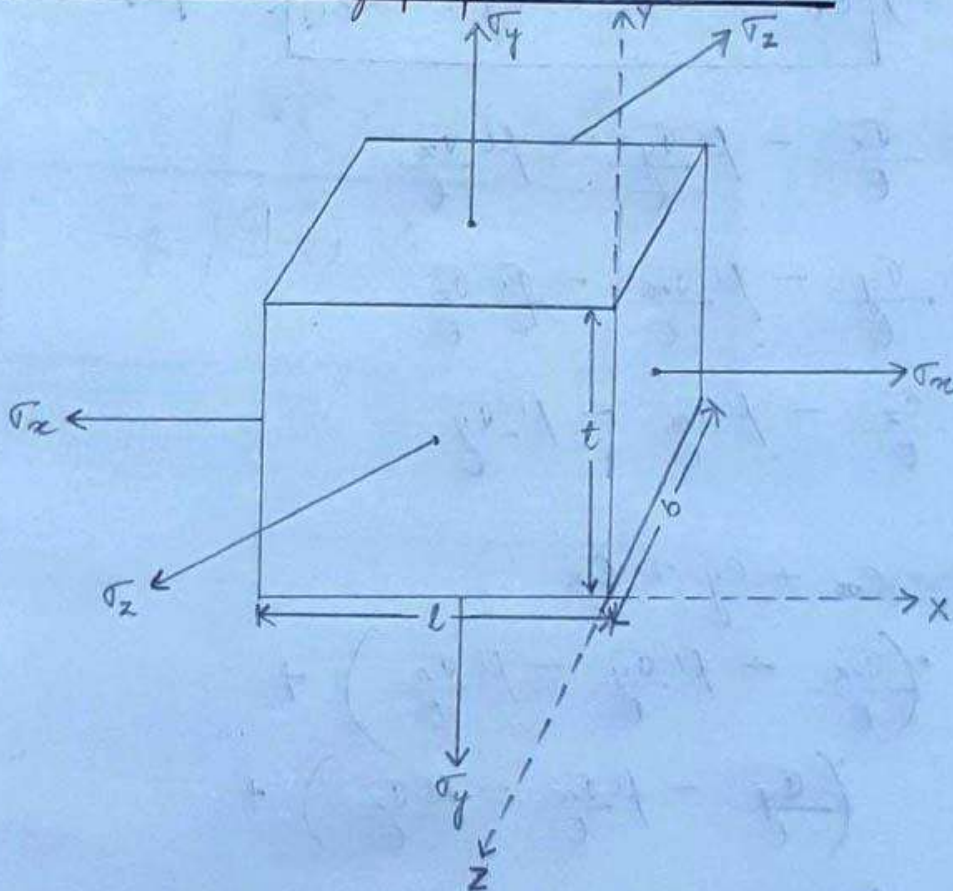
Now put the value of  $e_x$ ,  $e_y$  and  $e_z$

$$e_v = \frac{\sigma_x}{E} - \mu \frac{\sigma_x}{E} - \mu \frac{\sigma_x}{E}$$

$$\Rightarrow e_v = \frac{\sigma_x}{E} - 2\mu \frac{\sigma_x}{E}$$

$$\Rightarrow e_v = \frac{\sigma_x}{E} (1 - 2\mu) \quad \begin{matrix} *** \\ ** \end{matrix}$$

② Volumetric strain of a rectangular block subjected to three mutually perpendicular stress:



Let  $l$ ,  $b$ ,  $t$  are the length, breadth and thickness of the rectangular block.

$\sigma_x$ ,  $\sigma_y$ ,  $\sigma_z$  are the three stresses act in the direction of  $x$ ,  $y$ ,  $z$ .

We know that,

$$V = l \times b \times t$$

$$\text{and } \frac{dV}{V} = \frac{dl}{l} + \frac{db}{b} + \frac{dt}{t}$$

$$\Rightarrow \boxed{e_v = e_x + e_y + e_z}$$

$$\bullet e_x = \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} - \mu \frac{\sigma_z}{E}$$

$$\bullet e_y = \frac{\sigma_y}{E} - \mu \frac{\sigma_x}{E} - \mu \frac{\sigma_z}{E}$$

$$\bullet e_z = \frac{\sigma_z}{E} - \mu \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E}$$

$$\therefore e_v = e_x + e_y + e_z$$

$$= \left( \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} - \mu \frac{\sigma_z}{E} \right) +$$

$$\left( \frac{\sigma_y}{E} - \mu \frac{\sigma_x}{E} - \mu \frac{\sigma_z}{E} \right) +$$

$$\left( \frac{\sigma_z}{E} - \mu \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} \right)$$



$$\Rightarrow e_v = \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} - \mu \frac{\sigma_z}{E} + \frac{\sigma_y}{E} - \mu \frac{\sigma_x}{E} - \mu \frac{\sigma_z}{E} + \frac{\sigma_z}{E} - \mu \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E}$$

$$\Rightarrow e_v = \frac{1}{E} \left[ \sigma_x - \mu \sigma_y - \mu \sigma_z + \sigma_y - \mu \sigma_x - \mu \sigma_z - \sigma_z - \mu \sigma_x - \mu \sigma_y \right]$$

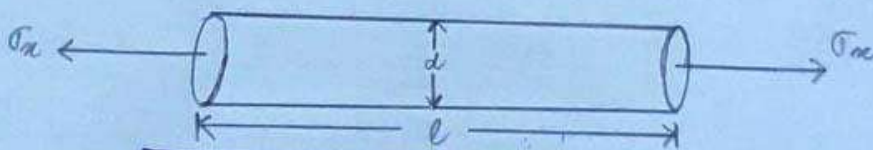
$$\Rightarrow e_v = \frac{1}{E} \left[ \sigma_x - \mu \sigma_x - \mu \sigma_x + \sigma_y - \mu \sigma_y - \mu \sigma_y + \sigma_z - \mu \sigma_z - \mu \sigma_z \right]$$

$$\Rightarrow e_v = \frac{1}{E} \left[ \sigma_x - 2\mu \sigma_x + \sigma_y - 2\mu \sigma_y + \sigma_z - 2\mu \sigma_z \right]$$

$$\Rightarrow e_v = \frac{1}{E} \left[ \sigma_x(1-2\mu) + \sigma_y(1-2\mu) + \sigma_z(1-2\mu) \right]$$

$$\Rightarrow e_v = \frac{1}{E} \left[ (1-2\mu)(\sigma_x + \sigma_y + \sigma_z) \right]$$

③ Volumetric strain for a circular Rod:



$$e_v = 2e_d + e_l$$

where,  $e_v$  = volumetric strain

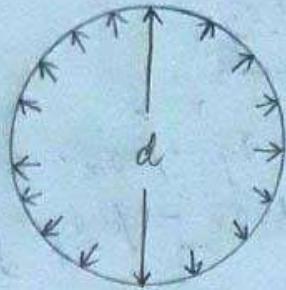
$e_d$  = lateral strain

$e_l$  = longitudinal strain

***Complex  
Stress and  
Strain .***



④ Volumetric strain for a sphere:

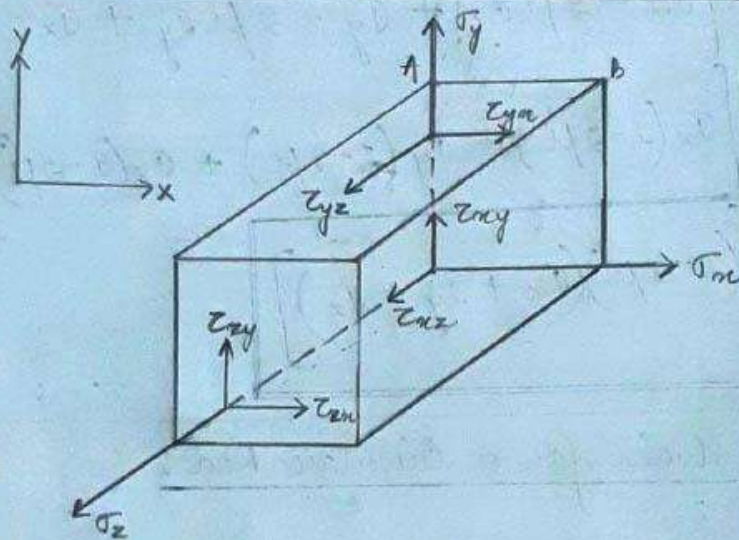


$$e_v = 3e_d$$

CHAPTER - 3

29.11.2021

COMPLEX STRAIN AND STRESS:



3-D representation of stresses  
(or)

3-dimensional stress system